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A Problem with Structured Propositions

Abstract: The paper shows that the paradox of the totality of propositions rests on assumptions characteristic of some theories of structured contents. Thus – it is shown that the threat of possible paradoxes, contrary to the widespread opinion, is not solely connected with possible-world theories of content. Moreover, possible-world theories - due to the rejection of structural constraints put on contents - can avoid the paradox in quite an elementary way.

Key words: structured propositions, Russell-Kaplan paradox

It has been claimed by many authors¹ that, among various accounts of propositions, those putting no constraints on internal structure of content (e.g. conceiving contents as sets of possible worlds) are much more problematic than those treating contents as structured entities. One of several problems² mentioned in the philosophical literature (e.g. Lewis 1986: 104-108, Kaplan 1995) is the threat of inconsistency that results from the possibility of constructing, within unstructured approaches, counterparts of set-theoretical paradoxes. The minimal moral to draw from this eventuality, it has been claimed, is that philosophers sympathetic towards them ought to rethink the foundations of their theory – maybe even explicitly admit that it is committed to a particular variant of set theory or intensional logic (cf. Oksanen 1999, Cocchiarella 2000).

Nobody, to my knowledge, pointed out, however, that a similar problem may arise for theories of propositions that individuate contents in a way that relies too closely on the structure of sentences. Below I argue that this is a serious mistake. The argument that shows this to be the case was formulated in 1984 by Patrick Grim (Grim 1984). Although Grim's

¹ Robert Stalnaker being a prominent exception.

² Of which the most infamous is still that of identifying propositions expressed by logically equivalent sentences.

reasoning was intended primarily to address the issue of the existence of the set of all truths (he also mentions its possible application to ersatz theories of possible worlds), it relies on some assumptions concerning the structured nature of propositions, i.e. theories engendering the paradox are precisely those committed to those assumptions. Slightly modified,³ Grim's argument goes as follows. Let \mathbf{P} be the set of all propositions:

$$\mathbf{P} = \{P_1, P_2, P_3, \dots\}$$

and $2^{\mathbf{P}}$ be its power set:

$$2^{\mathbf{P}} = \{ \emptyset, \{P_1\}, \{P_2\}, \{P_3\}, \dots \}$$

We may assign a true *sentence* to each element of $2^{\mathbf{P}}$ as follows:

$$\begin{aligned} \emptyset &\rightarrow "P_1 \notin \emptyset" \\ \{P_1\} &\rightarrow "P_1 \in \{P_1\}" \\ \{P_2\} &\rightarrow "P_1 \notin \{P_2\}" \\ \{P_3\} &\rightarrow "P_1 \notin \{P_3\}" \\ &\cdot \\ &\cdot \\ &\cdot \\ \{P_1, P_2\} &\rightarrow "P_1 \in \{P_1, P_2\}" \\ \{P_2, P_3\} &\rightarrow "P_1 \notin \{P_2, P_3\}" \\ &\cdot \\ &\cdot \\ &\cdot \end{aligned}$$

i.e. for each element x of $2^{\mathbf{P}}$ we assign a sentence of the form " $P_1 \in x$ " if $P_1 \in x$ or of the form " $P_1 \notin x$ " if $P_1 \notin x$ (the choice of this particular element of \mathbf{P} is irrelevant). Other lists, similar to this one, are also possible.

³ Grim starts with the set of all truths while we are starting with the set of all sentences/propositions – not excluding the possibility that it contains also false ones. We are also formulating the argument in terms of sentences and (possibly) their contents to make its structural assumptions explicit.

For example, one may use the identity sign instead of set-membership and its negation - the result will be a list of sentences like “ $P_1 = \emptyset$ ”, “ $P_1 = \{P_1\}$ ”, “ $P_1 = \{P_1, P_2\}$ ” etc. (false, if the axiom of foundation is assumed).

Now, if *each sentence on our list expresses a different proposition*, we will be able to show that there are at least as many propositions as subsets of the power set of the class of all propositions. This, of course, contradicts the well known theorem of Cantor. Note that the assumption to the effect that each sentence on our list expresses a distinct proposition (call it (A)) is essential to our argument – if someone, for example, takes propositions to be sets of possible worlds, then her move is to deny precisely this claim by noting that all sentences on our list really express a *single necessary proposition* (at least, if you believe that all sentences like “ $x \in \{x\}$ ” etc. are necessary).

In recent years Jeffrey King (King 1995, 1996, 2007) has defended a structured theory of propositions (he calls it “the new account of structured propositions”) which entails that all sentences on our list differ in contents. One of the claims of his theory is: “Sentences with different syntactic structures at LF ipso facto express different propositions” (King 2007: 95). (LF is the level of syntax that is the output to semantics). Moreover, the relation that connects elements of the structured proposition is the result of composing sentential relations abstracted from LF of some particular sentence with semantic relations holding between terminal elements of this sentence and their semantic values. This results, among other things, in consequences like:

(...) sentences “Laura is happy and Scott is sad” and “Scott is sad and Laura is happy” express different propositions; similarly for “ $1 = 2$ ” and “ $2 = 1$ ”. In both cases, the propositions expressed by the sentence pairs will have different constituents occurring at corresponding positions (terminal nodes) in the propositions. (King 2007: 95)

To see that King's view entails (A) note that all sentences on our list have one of two general forms (a) “ $P_1 \in x$ ” or (b) “ $P_1 \notin x$ ” and that our list could be divided into two sub-lists – each containing either only (a)-sentences

or (b)-sentences.⁴ Each sentence on an appropriate sub-list will differ only with respect to a particular instantiation of the variable x . Now, x takes as particular instantiations various set-denoting expressions (e.g. “ $\{P_1, P_2\}$ ”) and there are generally three approaches that one may adopt towards them:

(1) One may claim – though this is not very likely – that they are like proper names, i.e. singular terms without relevant internal structure. In such a case, since they denote distinct objects, ($\{P_1, P_2\}$ is certainly distinct from $\{P_2, P_3\}$ etc.), they have different semantic values, which means that all propositions on our list would have “different constituents occurring at corresponding positions (terminal nodes) in the propositions”.

(2) One may – this is more likely – take them to be abbreviations for complex abstract terms like “ $\{x: x \neq x\}$ ” (which we abbreviate as “ \emptyset ”) or “ $\{x: x = P_2\} \cup \{x: x = P_3\}$ ” (which we abbreviate as “ $\{P_2, P_3\}$ ”). That would, of course, not matter (when compared with (1)) insofar as we are assuming that a difference in reference implies a difference in semantic value. But, even if we reject this assumption (I do not think that King is committed to that), we will arrive at the very same conclusion due to the fact that the syntactic dissimilarities between our propositions would be quite noticeable (sometimes as huge as *syntactic* distinctions between “ $\{x: x \neq x\}$ ”, “ $\{x: x = P_2\} \cup \{x: x = P_3\}$ ” and “ $\{x: x = P_1\} \cup \{x: x = P_2\} \cup \{x: x = P_3\} \cup \{x: x = P_4\} \cup \{x: x = P_5\} \cup \{x: x = P_6\}$ ”, for example).

(3) One may claim that they are incomplete symbols that can be eliminated, when placed in the sentential context, by means of contextual definitions; for example, “ $P_1 \in \{P_1\}$ ” could be replaced by “ $\exists x (P_1 \in x) \ \& \ \forall z (z \in x \ \& \ z = P_1)$ ”. Again, we may repeat here the line of reasoning from previous points: even if one rejected the view that a difference in reference implies a difference in semantic value, the syntactic differences between propositions cannot be ignored; e.g., “ $(\exists x$

⁴ Note that our reasoning may be simplified, if one used identity sign instead of set-membership. In such a case we would have only one list with different arguments on the right side of identity statement.

$(P_1 \in x) \ \& \ \forall z (z \in x \ \& \ z = P_1)$ ” and “ $\exists x (P_2 \in x) \ \& \ \forall z (z \in x \ \& \ z = P_2)$ ” would differ exactly due to the presence of distinct singular terms on corresponding syntactical places.

Furthermore, no sentence from the (a)-list would express the same proposition as the sentence from (b)-list (and *vice versa*): (b)-sentences are negations while (a)-sentences are not. Thus, “the new account of structured propositions” cannot consistently speak about classes of all propositions, all false propositions falsehoods, all true propositions etc. The same can be said of some of its predecessors (cf. Ajdukiewicz 1967a, 1967b). I must admit that while I see the reason why one may want to ignore the problem in the first case (the class of all propositions) I agree with Grim’s opinion that the reasoning derives its philosophical significance from other cases (especially the case of the set of all true propositions). On the other hand, the problem is avoided by theories that are more liberal in the way they individuate contents*.

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* I would like to extend my thanks to Witold Hensel, Tomasz Puczyłowski, Mieszko Tałasiewicz and Bartosz Wcisło for valuable comments on an earlier version of this paper.

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